Review: Dynamical systems metrics

1. Definition

Based on dynamical system theory and extremal theory. Here we describe this dynamical system via 3 metrics, including local dimension *d*, local persistence θ-1 and co-cooccurrence ratio α.

Supposed a trajectory X(t), which denotes a variable’s lat-lon maps. We measure the dynamical property of a specific point ζvia calculating a Euclidean distance between ζ and a point on the trajectory, then the logarithmic distance returns as: . Then, we focus on the analogs of ζ on the trajectory: given a threshold *S* being the 98th quantile of g(x(t), ζ), we can extract the exceedances like . Here *u* only takes the positive values. The cumulative probability distribution *F*(*u, ζ*) follows the Generalized Pareto Distribution function (extremal theory).

* 1. Local dimensions (spatial information)

This parameter represents the degree of freedom near ζ. It can be easily gotten by , where σ equals the average of *u.* Applying this parameter help us identify the spatial information in variable.

* 1. Local persistence (temporal information)

This parameter depicts how long it stays within the point ζ. It is estimated by maximum likelihood estimation as:

here , N is the number of *u, ,* here j denotes the numerical order and *q* equals

A high value of θ means the residence time of the system around ζ is shorter. So local persistence is defined as θ-1. The residence time of system around ζ is proportional to local persistence.

1.3 co-occurrence ratio

Here co-occurrence ratio α is defined as:

where ν[-] is the number of events which realize the condition of -. Specifically, the two trajectories should be normalized via their norm, like x/||x|| and y/||y|| (|| || is the average root mean square norm).

This parameter can identify correlation among variables.